Guía de Matemáticas III, Otoño 2023. Segundo Parcial

## Subespacios Vectoriales

1. Determine si los siguientes conjuntos son sub-espacios vectoriales:

## Combinación Lineal

1. Dados los vectores , y . Exprese los siguientes vectores como combinaciones lineales de , y .

$$
\left[\begin{matrix}2 & 1 & 3 & 5\\1 & -1 & 2 & 5\\4 & 3 & 5 & 9\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & 9\\1 & -1 & 2 & 5\\2 & 1 & 3 & 5\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 } \\
\left[\begin{matrix}4 & 3 & 5 & 9\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{11}{4}\\2 & 1 & 3 & 5\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & 9\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{11}{4}\\0 & - \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} } \\
\left[\begin{matrix}4 & 3 & 5 & 9\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{11}{4}\\0 & - \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\left[\begin{matrix}4 & 3 & 5 & 9\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{11}{4}\\0 & 0 & \frac{2}{7} & - \frac{2}{7}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} } \\
\left[\begin{matrix}4 & 3 & 5 & 9\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{11}{4}\\0 & 0 & \frac{2}{7} & - \frac{2}{7}\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & 9\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{11}{4}\\0 & 0 & 1 & -1\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 } \\
\left[\begin{matrix}4 & 3 & 5 & 9\\0 & - \frac{7}{4} & 0 & \frac{7}{2}\\0 & 0 & 1 & -1\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\left[\begin{matrix}4 & 3 & 0 & 14\\0 & - \frac{7}{4} & 0 & \frac{7}{2}\\0 & 0 & 1 & -1\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} } \\
\left[\begin{matrix}4 & 3 & 0 & 14\\0 & 1 & 0 & -2\\0 & 0 & 1 & -1\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\left[\begin{matrix}4 & 0 & 0 & 20\\0 & 1 & 0 & -2\\0 & 0 & 1 & -1\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & 5\\0 & 1 & 0 & -2\\0 & 0 & 1 & -1\end{matrix}\right]
$$

$$
\left[\begin{matrix}2 & 1 & 3 & 2\\1 & -1 & 2 & 0\\4 & 3 & 5 & 6\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & 6\\1 & -1 & 2 & 0\\2 & 1 & 3 & 2\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\2 & 1 & 3 & 2\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\\
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & - \frac{1}{2} & \frac{1}{2} & -1\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & - \frac{1}{2} & \frac{1}{2} & -1\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\\
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & 0 & \frac{2}{7} & - \frac{4}{7}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & 0 & \frac{2}{7} & - \frac{4}{7}\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\\
\left[\begin{matrix}4 & 3 & 0 & 16\\0 & - \frac{7}{4} & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} }
\left[\begin{matrix}4 & 3 & 0 & 16\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\\
\left[\begin{matrix}4 & 0 & 0 & 16\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & 4\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
$$

$$
\left[\begin{matrix}2 & 1 & 3 & 2\\1 & -1 & 2 & 2\\4 & 3 & 5 & 3\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & 3\\1 & -1 & 2 & 2\\2 & 1 & 3 & 2\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & 3\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{5}{4}\\2 & 1 & 3 & 2\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\\
\left[\begin{matrix}4 & 3 & 5 & 3\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{5}{4}\\0 & - \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}4 & 3 & 5 & 3\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{5}{4}\\0 & - \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\\
\left[\begin{matrix}4 & 3 & 5 & 3\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{5}{4}\\0 & 0 & \frac{2}{7} & \frac{1}{7}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}4 & 3 & 5 & 3\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{5}{4}\\0 & 0 & \frac{2}{7} & \frac{1}{7}\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & 3\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{5}{4}\\0 & 0 & 1 & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 }
\left[\begin{matrix}4 & 3 & 5 & 3\\0 & - \frac{7}{4} & 0 & \frac{7}{8}\\0 & 0 & 1 & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\\
\left[\begin{matrix}4 & 3 & 0 & \frac{1}{2}\\0 & - \frac{7}{4} & 0 & \frac{7}{8}\\0 & 0 & 1 & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} }
\left[\begin{matrix}4 & 3 & 0 & \frac{1}{2}\\0 & 1 & 0 & - \frac{1}{2}\\0 & 0 & 1 & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\\
\left[\begin{matrix}4 & 0 & 0 & 2\\0 & 1 & 0 & - \frac{1}{2}\\0 & 0 & 1 & \frac{1}{2}\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & \frac{1}{2}\\0 & 1 & 0 & - \frac{1}{2}\\0 & 0 & 1 & \frac{1}{2}\end{matrix}\right]
$$

$$
\left[\begin{matrix}2 & 1 & 3 & -1.0\\1 & -1 & 2 & 3.0\\4 & 3 & 5 & 0.5\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & 0.5\\1 & -1 & 2 & 3.0\\2 & 1 & 3 & -1.0\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & 0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 2.875\\2 & 1 & 3 & -1.0\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\\
\left[\begin{matrix}4 & 3 & 5 & 0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 2.875\\0 & - \frac{1}{2} & \frac{1}{2} & -1.25\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}4 & 3 & 5 & 0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 2.875\\0 & - \frac{1}{2} & \frac{1}{2} & -1.25\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\\
\left[\begin{matrix}4 & 3 & 5 & 0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 2.875\\0 & 0 & \frac{2}{7} & -2.07142857142857\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}4 & 3 & 5 & 0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 2.875\\0 & 0 & \frac{2}{7} & -2.07142857142857\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & 0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 2.875\\0 & 0 & 1 & -7.25\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 }
\left[\begin{matrix}4 & 3 & 5 & 0.5\\0 & - \frac{7}{4} & 0 & 8.3125\\0 & 0 & 1 & -7.25\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\\
\left[\begin{matrix}4 & 3 & 0 & 36.75\\0 & - \frac{7}{4} & 0 & 8.3125\\0 & 0 & 1 & -7.25\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} }
\left[\begin{matrix}4 & 3 & 0 & 36.75\\0 & 1 & 0 & -4.75\\0 & 0 & 1 & -7.25\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\\
\left[\begin{matrix}4 & 0 & 0 & 51.0\\0 & 1 & 0 & -4.75\\0 & 0 & 1 & -7.25\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & 12.75\\0 & 1 & 0 & -4.75\\0 & 0 & 1 & -7.25\end{matrix}\right]
$$

1. Si , y . Exprese los siguientes polinomios como una combinación de , y .

$$
\left[\begin{matrix}4 & 3 & 5 & -5\\1 & -1 & 2 & 9\\2 & 1 & 3 & 5\end{matrix}\right]
\underrightarrow{ R\_{1} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & -5\\1 & -1 & 2 & 9\\2 & 1 & 3 & 5\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & -5\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{41}{4}\\2 & 1 & 3 & 5\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\\
\left[\begin{matrix}4 & 3 & 5 & -5\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{41}{4}\\0 & - \frac{1}{2} & \frac{1}{2} & \frac{15}{2}\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}4 & 3 & 5 & -5\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{41}{4}\\0 & - \frac{1}{2} & \frac{1}{2} & \frac{15}{2}\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\\
\left[\begin{matrix}4 & 3 & 5 & -5\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{41}{4}\\0 & 0 & \frac{2}{7} & \frac{32}{7}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}4 & 3 & 5 & -5\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{41}{4}\\0 & 0 & \frac{2}{7} & \frac{32}{7}\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & -5\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{41}{4}\\0 & 0 & 1 & 16\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 }
\left[\begin{matrix}4 & 3 & 5 & -5\\0 & - \frac{7}{4} & 0 & - \frac{7}{4}\\0 & 0 & 1 & 16\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\\
\left[\begin{matrix}4 & 3 & 0 & -85\\0 & - \frac{7}{4} & 0 & - \frac{7}{4}\\0 & 0 & 1 & 16\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} }
\left[\begin{matrix}4 & 3 & 0 & -85\\0 & 1 & 0 & 1\\0 & 0 & 1 & 16\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\\
\left[\begin{matrix}4 & 0 & 0 & -88\\0 & 1 & 0 & 1\\0 & 0 & 1 & 16\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & -22\\0 & 1 & 0 & 1\\0 & 0 & 1 & 16\end{matrix}\right]
$$

$$
\left[\begin{matrix}4 & 3 & 5 & 6\\1 & -1 & 2 & 0\\2 & 1 & 3 & 2\end{matrix}\right]
\underrightarrow{ R\_{1} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & 6\\1 & -1 & 2 & 0\\2 & 1 & 3 & 2\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\2 & 1 & 3 & 2\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\\
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & - \frac{1}{2} & \frac{1}{2} & -1\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & - \frac{1}{2} & \frac{1}{2} & -1\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\\
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & 0 & \frac{2}{7} & - \frac{4}{7}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & 0 & \frac{2}{7} & - \frac{4}{7}\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & \frac{3}{4} & - \frac{3}{2}\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 }
\left[\begin{matrix}4 & 3 & 5 & 6\\0 & - \frac{7}{4} & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\\
\left[\begin{matrix}4 & 3 & 0 & 16\\0 & - \frac{7}{4} & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} }
\left[\begin{matrix}4 & 3 & 0 & 16\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\\
\left[\begin{matrix}4 & 0 & 0 & 16\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & 4\\0 & 1 & 0 & 0\\0 & 0 & 1 & -2\end{matrix}\right]
$$

$$
\left[\begin{matrix}4 & 3 & 5 & -0.5\\1 & -1 & 2 & 1.0\\2 & 1 & 3 & 3.0\end{matrix}\right]
\underrightarrow{ R\_{1} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & -0.5\\1 & -1 & 2 & 1.0\\2 & 1 & 3 & 3.0\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & -0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 1.125\\2 & 1 & 3 & 3.0\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\\
\left[\begin{matrix}4 & 3 & 5 & -0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 1.125\\0 & - \frac{1}{2} & \frac{1}{2} & 3.25\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}4 & 3 & 5 & -0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 1.125\\0 & - \frac{1}{2} & \frac{1}{2} & 3.25\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\\
\left[\begin{matrix}4 & 3 & 5 & -0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 1.125\\0 & 0 & \frac{2}{7} & 2.92857142857143\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}4 & 3 & 5 & -0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 1.125\\0 & 0 & \frac{2}{7} & 2.92857142857143\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & -0.5\\0 & - \frac{7}{4} & \frac{3}{4} & 1.125\\0 & 0 & 1 & 10.25\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 }
\left[\begin{matrix}4 & 3 & 5 & -0.5\\0 & - \frac{7}{4} & 0 & -6.5625\\0 & 0 & 1 & 10.25\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\\
\left[\begin{matrix}4 & 3 & 0 & -51.75\\0 & - \frac{7}{4} & 0 & -6.5625\\0 & 0 & 1 & 10.25\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} }
\left[\begin{matrix}4 & 3 & 0 & -51.75\\0 & 1 & 0 & 3.75\\0 & 0 & 1 & 10.25\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\\
\left[\begin{matrix}4 & 0 & 0 & -63.0\\0 & 1 & 0 & 3.75\\0 & 0 & 1 & 10.25\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & -15.75\\0 & 1 & 0 & 3.75\\0 & 0 & 1 & 10.25\end{matrix}\right]
$$

$$
\left[\begin{matrix}4 & 3 & 5 & -2\\1 & -1 & 2 & 7\\2 & 1 & 3 & 1\end{matrix}\right]
\underrightarrow{ R\_{1} \leftrightarrow R\_{1} }
\left[\begin{matrix}4 & 3 & 5 & -2\\1 & -1 & 2 & 7\\2 & 1 & 3 & 1\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{4})R\_1 }
\left[\begin{matrix}4 & 3 & 5 & -2\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{15}{2}\\2 & 1 & 3 & 1\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{1}{2})R\_1 }
\\
\left[\begin{matrix}4 & 3 & 5 & -2\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{15}{2}\\0 & - \frac{1}{2} & \frac{1}{2} & 2\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}4 & 3 & 5 & -2\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{15}{2}\\0 & - \frac{1}{2} & \frac{1}{2} & 2\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{7})R\_2 }
\\
\left[\begin{matrix}4 & 3 & 5 & -2\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{15}{2}\\0 & 0 & \frac{2}{7} & - \frac{1}{7}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}4 & 3 & 5 & -2\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{15}{2}\\0 & 0 & \frac{2}{7} & - \frac{1}{7}\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{2}{7}} }
\left[\begin{matrix}4 & 3 & 5 & -2\\0 & - \frac{7}{4} & \frac{3}{4} & \frac{15}{2}\\0 & 0 & 1 & - \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{3}{4})R\_3 }
\left[\begin{matrix}4 & 3 & 5 & -2\\0 & - \frac{7}{4} & 0 & \frac{63}{8}\\0 & 0 & 1 & - \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_1 + (-5)R\_3 }
\\
\left[\begin{matrix}4 & 3 & 0 & \frac{1}{2}\\0 & - \frac{7}{4} & 0 & \frac{63}{8}\\0 & 0 & 1 & - \frac{1}{2}\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{7}{4}} }
\left[\begin{matrix}4 & 3 & 0 & \frac{1}{2}\\0 & 1 & 0 & - \frac{9}{2}\\0 & 0 & 1 & - \frac{1}{2}\end{matrix}\right]
\underrightarrow{ R\_1 + (-3)R\_2 }
\\
\left[\begin{matrix}4 & 0 & 0 & 14\\0 & 1 & 0 & - \frac{9}{2}\\0 & 0 & 1 & - \frac{1}{2}\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{4} }
\left[\begin{matrix}1 & 0 & 0 & \frac{7}{2}\\0 & 1 & 0 & - \frac{9}{2}\\0 & 0 & 1 & - \frac{1}{2}\end{matrix}\right]
$$

1. Escriba a como una combinación lineal del conjunto de vectores A.

$$
\left[\begin{matrix}-2 & 4 & 3 & -1\\-1 & -1 & 1 & -2\\-5 & -2 & -3 & 4\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{1} }
\left[\begin{matrix}-5 & -2 & -3 & 4\\-1 & -1 & 1 & -2\\-2 & 4 & 3 & -1\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{5})R\_1 }
\left[\begin{matrix}-5 & -2 & -3 & 4\\0 & - \frac{3}{5} & \frac{8}{5} & - \frac{14}{5}\\-2 & 4 & 3 & -1\end{matrix}\right]
\underrightarrow{ R\_3 + (- \frac{2}{5})R\_1 }
\\
\left[\begin{matrix}-5 & -2 & -3 & 4\\0 & - \frac{3}{5} & \frac{8}{5} & - \frac{14}{5}\\0 & \frac{24}{5} & \frac{21}{5} & - \frac{13}{5}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{2} }
\left[\begin{matrix}-5 & -2 & -3 & 4\\0 & \frac{24}{5} & \frac{21}{5} & - \frac{13}{5}\\0 & - \frac{3}{5} & \frac{8}{5} & - \frac{14}{5}\end{matrix}\right]
\underrightarrow{ R\_3 + (\frac{1}{8})R\_2 }
\\
\left[\begin{matrix}-5 & -2 & -3 & 4\\0 & \frac{24}{5} & \frac{21}{5} & - \frac{13}{5}\\0 & 0 & \frac{17}{8} & - \frac{25}{8}\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}-5 & -2 & -3 & 4\\0 & \frac{24}{5} & \frac{21}{5} & - \frac{13}{5}\\0 & 0 & \frac{17}{8} & - \frac{25}{8}\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{\frac{17}{8}} }
\left[\begin{matrix}-5 & -2 & -3 & 4\\0 & \frac{24}{5} & \frac{21}{5} & - \frac{13}{5}\\0 & 0 & 1 & - \frac{25}{17}\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{21}{5})R\_3 }
\left[\begin{matrix}-5 & -2 & -3 & 4\\0 & \frac{24}{5} & 0 & \frac{304}{85}\\0 & 0 & 1 & - \frac{25}{17}\end{matrix}\right]
\underrightarrow{ R\_1 + (3)R\_3 }
\\
\left[\begin{matrix}-5 & -2 & 0 & - \frac{7}{17}\\0 & \frac{24}{5} & 0 & \frac{304}{85}\\0 & 0 & 1 & - \frac{25}{17}\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{\frac{24}{5}} }
\left[\begin{matrix}-5 & -2 & 0 & - \frac{7}{17}\\0 & 1 & 0 & \frac{38}{51}\\0 & 0 & 1 & - \frac{25}{17}\end{matrix}\right]
\underrightarrow{ R\_1 + (2)R\_2 }
\\
\left[\begin{matrix}-5 & 0 & 0 & \frac{55}{51}\\0 & 1 & 0 & \frac{38}{51}\\0 & 0 & 1 & - \frac{25}{17}\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{-5} }
\left[\begin{matrix}1 & 0 & 0 & - \frac{11}{51}\\0 & 1 & 0 & \frac{38}{51}\\0 & 0 & 1 & - \frac{25}{17}\end{matrix}\right]
$$

1. :

$$
\begin{bmatrix} 1 & 1 &1 &1 & -1 \\ 0 & 0& -1 & 5 & 2 \\ -1 & 1 &-1 & -1 & 0 \end{bmatrix} \underrightarrow{R\_3 \rightarrow R\_1 + R\_3} \begin{bmatrix}1 & 1&1&1&-1\\0&0&-1&5&2\\0&2&0&0&-1\end{bmatrix} \underrightarrow{R\_2 \leftrightarrow R\_3} \\ \begin{bmatrix}1&0&1&1&-\frac{1}{2} \\ 0 & 1&0&0&-\frac{1}{2} \\ 0 & 0&-1&5&2\end{bmatrix} \underrightarrow{-R\_3} \begin{bmatrix}1&0&1&1&-\frac{1}{2}\\0&1&0&0&-\frac{1}{2}\\0&0&1&5&-2\end{bmatrix} \rightarrow{R\_1 \rightarrow -R\_3 + R\_1} \\ \begin{bmatrix}1&0&0&6&\frac{3}{2} \\ 0&1&0&0&-\frac{1}{2}\\0&0&1&5&-2\end{bmatrix} \therefore c\_1 = \frac{3}{2}, c\_2 = -\frac{1}{2}, c\_3 = -2, c\_4 = 0
$$

## Vectores Linealmente Independientes y Dependientes

1. Determine los valores de para que el conjunto sea lineal-mente independiente.
2. Sea el espacio vectorial de todas las funciones con valore real definidas sobre la recta real completa. ¿Cuáles de los siguientes conjuntos de vectores en son lineal-mente dependientes?
3. Nos da un sistema incosistente, por lo tanto es linealmente dependiente.

$$
\begin{bmatrix}x & \cos{x} & 0 \\ 1 & -\sin{x} & 0\end{bmatrix} \underrightarrow{R\_1 \leftrightarrow R\_2} \begin{bmatrix}1 & -\sin{x} & 0 \\ x & \cos{x} & 0\end{bmatrix} \underrightarrow{-xR\_1 + R\_2} \begin{bmatrix}1&\sin{x}&0 \\ 0 & \cos{x} +x\sin{x}&0\end{bmatrix}\ \underrightarrow{\frac{1}{\cos{x} + x\sin{x}}}R\_2 \\ \begin{bmatrix}1 & -\sin{x} & 0\\ 0&1&0\end{bmatrix} \underrightarrow{\sin{x}R\_2 + R\_1} \begin{bmatrix}1&0&0\\0&1&0\end{bmatrix} \therefore C\_1 = C\_2 = 0 \therefore \text{ Solución consistente, por lo tanto es linealmente independiente.}
$$

$$
\begin{bmatrix}0\\0\\0\end{bmatrix} \rightarrow \begin{bmatrix}1\\\sin{x}\\\sin{2x}\end{bmatrix} \therefore \begin{bmatrix}1&\sin{x}&\sin{2x} &0\\ 0 & \cos{x} & 2\cos{2x} & 0 \\ 0 & -\sin{x} & -4\sin{2x}&0\end{bmatrix} \underrightarrow{\frac{1}{\cos{x}}R\_2} \begin{bmatrix}1 & \sin{x} & \sin{2x} & 0 \\ 0&1&2\frac{\cos{2x}}{\cos{x}}&0\\0&-\sin{x}&-4\sin{2x}\end{bmatrix} \underrightarrow{-\sin{x}R\_2 + R\_1 \\ \sin{x}R-2 + R\_3} \\ \dots \\ \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix} \therefore \text{Solución consistente donde } c\_1 = c\_2 = c\_3 = 0 \text{ Por lo tanto es linealmente independiente}
$$

1. El sistema es linealmente independiente por ser un sistema consistente con soluciones infinitas.
2. Para que los valores de , las siguientes matrices son linealmente independientes de
3. Construya un conjunto de vectores tal que sean linealmente independientes y .

$$
V\_1^T V\_2 = \begin{bmatrix}1&0&0\end{bmatrix} \begin{bmatrix}0\\1\\0\end{bmatrix} = 0 \\ V\_2^TV\_3 = \begin{bmatrix}0&1&0\end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix} = 0 \\ \therefore \\ V\_1^TV\_2 = V\_2^TV\_1 = 0
$$

1. Sea . Demuestre que si , entonces es lineal-mente independiente.

## Bases y Cambios de Base

1. Determine una base para el espacio de funciones que satisface: .
2. Considera las bases y para R^2, donde:
3. Calcula la matriz de transición de hacia .

$$
\left[\begin{matrix}2 & 4 & 1 & -1\\2 & -1 & 3 & -1\end{matrix}\right]
\underrightarrow{ R\_{1} \leftrightarrow R\_{1} }
\left[\begin{matrix}2 & 4 & 1 & -1\\2 & -1 & 3 & -1\end{matrix}\right]
\underrightarrow{ R\_2 + (-1)R\_1 }
\left[\begin{matrix}2 & 4 & 1 & -1\\0 & -5 & 2 & 0\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}2 & 4 & 1 & -1\\0 & -5 & 2 & 0\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{-5} }
\left[\begin{matrix}2 & 4 & 1 & -1\\0 & 1 & - \frac{2}{5} & 0\end{matrix}\right]
\underrightarrow{ R\_1 + (-4)R\_2 }
\\
\left[\begin{matrix}2 & 0 & \frac{13}{5} & -1\\0 & 1 & - \frac{2}{5} & 0\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{2} }
\left[\begin{matrix}1 & 0 & \frac{13}{10} & - \frac{1}{2}\\0 & 1 & - \frac{2}{5} & 0\end{matrix}\right]
$$

1. Calcula la matriz de transición de hacia .

$$
\left[\begin{matrix}1 & -1 & 2 & 4\\3 & -1 & 2 & -1\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{1} }
\left[\begin{matrix}3 & -1 & 2 & -1\\1 & -1 & 2 & 4\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{3})R\_1 }
\left[\begin{matrix}3 & -1 & 2 & -1\\0 & - \frac{2}{3} & \frac{4}{3} & \frac{13}{3}\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}3 & -1 & 2 & -1\\0 & - \frac{2}{3} & \frac{4}{3} & \frac{13}{3}\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{2}{3}} }
\left[\begin{matrix}3 & -1 & 2 & -1\\0 & 1 & -2 & - \frac{13}{2}\end{matrix}\right]
\underrightarrow{ R\_1 + (1)R\_2 }
\\
\left[\begin{matrix}3 & 0 & 0 & - \frac{15}{2}\\0 & 1 & -2 & - \frac{13}{2}\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{3} }
\left[\begin{matrix}1 & 0 & 0 & - \frac{5}{2}\\0 & 1 & -2 & - \frac{13}{2}\end{matrix}\right]
$$

c) Dado el vector , calcula y .

$$
[W]\_B =
\left[\begin{matrix}2 & 4 & 3\\2 & -1 & -5\end{matrix}\right]
\underrightarrow{ R\_{1} \leftrightarrow R\_{1} }
\left[\begin{matrix}2 & 4 & 3\\2 & -1 & -5\end{matrix}\right]
\underrightarrow{ R\_2 + (-1)R\_1 }
\left[\begin{matrix}2 & 4 & 3\\0 & -5 & -8\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}2 & 4 & 3\\0 & -5 & -8\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{-5} }
\left[\begin{matrix}2 & 4 & 3\\0 & 1 & \frac{8}{5}\end{matrix}\right]
\underrightarrow{ R\_1 + (-4)R\_2 }
\\
\left[\begin{matrix}2 & 0 & - \frac{17}{5}\\0 & 1 & \frac{8}{5}\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{2} }
\left[\begin{matrix}1 & 0 & - \frac{17}{10}\\0 & 1 & \frac{8}{5}\end{matrix}\right]
$$

$$
[W]\_{B'} = \left[\begin{matrix}1 & -1 & 3\\3 & -1 & -5\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{1} }
\left[\begin{matrix}3 & -1 & -5\\1 & -1 & 3\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{3})R\_1 }
\left[\begin{matrix}3 & -1 & -5\\0 & - \frac{2}{3} & \frac{14}{3}\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}3 & -1 & -5\\0 & - \frac{2}{3} & \frac{14}{3}\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{- \frac{2}{3}} }
\left[\begin{matrix}3 & -1 & -5\\0 & 1 & -7\end{matrix}\right]
\underrightarrow{ R\_1 + (1)R\_2 }
\\
\left[\begin{matrix}3 & 0 & -12\\0 & 1 & -7\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{3} }
\left[\begin{matrix}1 & 0 & -4\\0 & 1 & -7\end{matrix}\right]
$$

1. Sean los polinomios , , realiza los siguientes ejercicios:

b) El conjunto , ¿forman una base? Justifica.

1. Dado el siguiente sistema de ecuaciones lineales homogéneo:

* Determina la base(si es que existe) del conjunto solución del problema.

$$
\begin{bmatrix}-1&3&1&0\\2&2&-1&0\\3&-1&-2&0\end{bmatrix} \underrightarrow{-R1}\begin{bmatrix}1&-3&1&0\\2&2&-1&0\\3&-1&-2&0\end{bmatrix} \underrightarrow{-2R\_1 + R\_2 \\ -3R\_1 + R\_3} \\ \begin{bmatrix}1&-3&-1&0\\0&8&1&0\\0&8&1&0\end{bmatrix} \underrightarrow{\frac{1}{8}R\_2}\begin{bmatrix}1&-3&-1&0\\0&1&\frac{1}{8}&0\\0&8&1&0\end{bmatrix}\underrightarrow{3R\_2 + R\_1 \\ -8R\_2 +R\_3} \\\begin{bmatrix}1 &0&-\frac{5}{6}&0\\0&1&\frac{1}{8}&0\\0&0&0&0\end{bmatrix} \therefore x = \frac{5}{8}z, y = -\frac{1}{8}z, z\in R
$$

$$
z\begin{bmatrix}\frac{5}{8} \\ -\frac{5}{8} \\1\end{bmatrix} = S \\ dim(S) =1
$$

1. Dados los vectores y bases siguientes:
2. Calcula .

$$
\left[\begin{matrix}1 & 0 & -1\\-1 & 1 & 3\end{matrix}\right]
\underrightarrow{ R\_{1} \leftrightarrow R\_{1} }
\left[\begin{matrix}1 & 0 & -1\\-1 & 1 & 3\end{matrix}\right]
\underrightarrow{ R\_2 + (1)R\_1 }
\left[\begin{matrix}1 & 0 & -1\\0 & 1 & 2\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}1 & 0 & -1\\0 & 1 & 2\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{1} }
\left[\begin{matrix}1 & 0 & -1\\0 & 1 & 2\end{matrix}\right]
\underrightarrow{ R\_1 + (0)R\_2 }
\\
\left[\begin{matrix}1 & 0 & -1\\0 & 1 & 2\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{1} }
\left[\begin{matrix}1 & 0 & -1\\0 & 1 & 2\end{matrix}\right]
$$

1. Calcula la matriz de transición de hacia .

$$
\left[\begin{matrix}-1 & 1 & 1 & 0\\-3 & 0 & -1 & 1\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{1} }
\left[\begin{matrix}-3 & 0 & -1 & 1\\-1 & 1 & 1 & 0\end{matrix}\right]
\underrightarrow{ R\_2 + (- \frac{1}{3})R\_1 }
\left[\begin{matrix}-3 & 0 & -1 & 1\\0 & 1 & \frac{4}{3} & - \frac{1}{3}\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{2} }
\left[\begin{matrix}-3 & 0 & -1 & 1\\0 & 1 & \frac{4}{3} & - \frac{1}{3}\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{1} }
\left[\begin{matrix}-3 & 0 & -1 & 1\\0 & 1 & \frac{4}{3} & - \frac{1}{3}\end{matrix}\right]
\underrightarrow{ R\_1 + (0)R\_2 }
\\
\left[\begin{matrix}-3 & 0 & -1 & 1\\0 & 1 & \frac{4}{3} & - \frac{1}{3}\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{-3} }
\left[\begin{matrix}1 & 0 & \frac{1}{3} & - \frac{1}{3}\\0 & 1 & \frac{4}{3} & - \frac{1}{3}\end{matrix}\right]
$$

1. Calcula utilizando la matriz de transición del inciso anterior.
2. Dado el siguiente conjunto de vectores: .
3. Determine si el conjunto genera a .

$$
\begin{bmatrix}\frac{1}{3} & -\frac{1}{3} & 0 & a\\ -7 & 0&-7&b\\0&2&\frac{1}{5}&c\end{bmatrix} \underrightarrow{2R\_1} \begin{bmatrix}1 & -\frac{2}{3} & 0 & 2a \\ -7 &0&-7&b\\0&2&\frac{1}{5}&c\end{bmatrix} \underrightarrow{7R1 + R\_2} \\\begin{bmatrix}1&-\frac{2}{3} & 0&2a\\0&-\frac{14}{3}&-7&14a+b\\0&2&\frac{1}{5}&c\end{bmatrix} \underrightarrow{-\frac{3}{14}R\_2} \begin{bmatrix}1&-\frac{2}{3}&0&2a\\0&1&\frac{2}{3}&-3a-\frac{3}{14}b\\0&2&\frac{1}{5}&c\end{bmatrix} \underrightarrow{\frac{2}{3}R\_2 + R1 \\-2R\_2 + R\_3}\\\begin{bmatrix}1&0&1&-2a-\frac{1}{7}b\\0&1&\frac{3}{2}&-3a-\frac{3}{14}b\\0&0&-\frac{14}{5}&6a+\frac{3}{7}b-2c\end{bmatrix} \underrightarrow{-\frac{5}{14}R\_3} \begin{bmatrix}1&0&1&-2a-\frac{1}{7}b\\0&1&\frac{3}{2}&-3a-\frac{3}{4}bb\\0&0&1&6a+\frac{3}{7}b-c\end{bmatrix} \underrightarrow{-R\_3 + R\_1 \\ - \frac{3}{2}R\_3 + R\_2} \begin{bmatrix}1&0&0&8a-\frac{4}{7}b +2c\\0&1&0&12a-\frac{6}{7}b+3c\\0&0&1&6s+\frac{3}{7}b-2c\end{bmatrix} \\ \therefore \text{Si es consistente, genera } R^3
$$

1. Genere un espacio vectorial de 3 elementos usando el conjunto de vectores.
2. Con los vectores, construya un sistema de ecuaciones lineales homogéneo y determine la base de las soluciones del sistema.

$$
\left[\begin{matrix}0.5 & -0.333333333333333 & 0 & 0\\-7.0 & 0 & -7.0 & 0\\0 & 2.0 & 0.2 & 0\end{matrix}\right]
\underrightarrow{ R\_{2} \leftrightarrow R\_{1} }
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0.5 & -0.333333333333333 & 0 & 0\\0 & 2.0 & 0.2 & 0\end{matrix}\right]
\underrightarrow{ R\_2 + (0.0714285714285714)R\_1 }
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0 & -0.333333333333333 & -0.5 & 0\\0 & 2.0 & 0.2 & 0\end{matrix}\right]
\underrightarrow{ R\_3 + (0)R\_1 }
\\
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0 & -0.333333333333333 & -0.5 & 0\\0 & 2.0 & 0.2 & 0\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{2} }
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0 & 2.0 & 0.2 & 0\\0 & -0.333333333333333 & -0.5 & 0\end{matrix}\right]
\underrightarrow{ R\_3 + (0.166666666666667)R\_2 }
\\
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0 & 2.0 & 0.2 & 0\\0 & 0 & -0.466666666666667 & 0\end{matrix}\right]
\underrightarrow{ R\_{3} \leftrightarrow R\_{3} }
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0 & 2.0 & 0.2 & 0\\0 & 0 & -0.466666666666667 & 0\end{matrix}\right]
\underrightarrow{ \frac{R\_3}{-0.466666666666667} }
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0 & 2.0 & 0.2 & 0\\0 & 0 & 1.0 & 0\end{matrix}\right]
\underrightarrow{ R\_2 + (-0.2)R\_3 }
\left[\begin{matrix}-7.0 & 0 & -7.0 & 0\\0 & 2.0 & 0 & 0\\0 & 0 & 1.0 & 0\end{matrix}\right]
\underrightarrow{ R\_1 + (7.0)R\_3 }
\\
\left[\begin{matrix}-7.0 & 0 & 0 & 0\\0 & 2.0 & 0 & 0\\0 & 0 & 1.0 & 0\end{matrix}\right]
\underrightarrow{ \frac{R\_2}{2.0} }
\left[\begin{matrix}-7.0 & 0 & 0 & 0\\0 & 1.0 & 0 & 0\\0 & 0 & 1.0 & 0\end{matrix}\right]
\underrightarrow{ R\_1 + (0)R\_2 }
\\
\left[\begin{matrix}-7.0 & 0 & 0 & 0\\0 & 1.0 & 0 & 0\\0 & 0 & 1.0 & 0\end{matrix}\right]
\underrightarrow{ \frac{R\_1}{-7.0} }
\left[\begin{matrix}1.0 & 0 & 0 & 0\\0 & 1.0 & 0 & 0\\0 & 0 & 1.0 & 0\end{matrix}\right]
$$

$$
\therefore \\
C\_1 = C\_2 = C\_3 = 0
$$

Sistema consistente, que genera a

1. Sea y determine la base , sabiendo que
2. Sea y , sabiendo que y , determine:
3. Matriz de transición de la base hacia
4. Matriz de transición de la base hacia .
5. Calcular las coordenadas de en términos de base . Calcule y . ¿Porué ?

## Rango y Nulidad de una matriz

1. Determine el valor de para que la matriz tenga

$$
M = \begin{bmatrix}1&-2&1&4\\3&-1&5&1\\5&-5&7&9\end{bmatrix} \underrightarrow{2R\_2 + R\_1 \\ -5R\_2 + R\_3} \begin{bmatrix}1&0&\frac{9}{5}&-\frac{2}{5}\\0&1&\frac{2}{5} & -\frac{11}{5} \\ 0&0&0&0\end{bmatrix} \\ \therefore \\ x = -\frac{9}{5}z + \frac{2}{5}w \\ y = -\frac{2}{5}z + \frac{11}{5}w \\ z \in R \\ w \in R \\ \therefore \\ z\begin{bmatrix}-\frac{9}{5} \\ -\frac{2}{5}\\1\\0\end{bmatrix}, w = \begin{bmatrix}\frac{2}{5}\\\frac{4}{5}\\0\\1\end{bmatrix} : V = 2
$$

1. Encuentre todos los valores posibles del rango de la matriz y , si es una variable.

K = -1

$$
\begin{bmatrix}1&2&-1\\-2&-4&2\\-1&-2&1\end{bmatrix} \underrightarrow{2R\_1 + R-2 \\ R\_1 + R\_2} \begin{bmatrix}1&2&-1\\0&0&0\\0&0&0\end{bmatrix} \\ \therefore \\ x = -2y + z \\ y \in R \\ z \in R\\ \therefore \\ \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-2y+z\\y\\z\end{bmatrix} \therefore y\begin{bmatrix}-2\\1\\0\end{bmatrix}, z\begin{bmatrix}1\\0\\1\end{bmatrix}\\V(A)= 2, P(A) = 1
$$

K = 2

$$
\begin{bmatrix}1&2&2\\-2&8&2\\2&-2&1\end{bmatrix} \underrightarrow{2R\_1 + R\_2 \\ -2R\_1 + R\_3} \begin{bmatrix}1&2&2\\0&12&8\\0&-6&-3\end{bmatrix} \underrightarrow{ \frac{1}{12}R\_2 } \\ \begin{bmatrix}1&2&2\\0&1&\frac{2}{3}\\0&-6&-3\end{bmatrix} \underrightarrow{ -2R\_2 + R\_1 \\ 6R\_2+R\_3 } \begin{bmatrix}1&0&\frac{2}{3}\\0&0&\frac{2}{3}\\0&0&1\end{bmatrix} \underrightarrow{-\frac{2}{3}R\_3 + R\_1 \\ - \frac{2}{3}R\_3 + R\_2} \\ \begin{bmatrix}1&0&1\\0&1&\frac{1}{2}\\0&0&1\end{bmatrix} \\ \therefore \\ x = 2 \\ y = -\frac{1}{2}z \\ z = 0 \\ \therefore \\ \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-2\\-\frac{1}{2}z \\0\end{bmatrix} = z\begin{bmatrix}-1\\-\frac{1}{2}\\0\end{bmatrix}
$$